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EDGE-TO-VERTEX DETOUR DISTANCE IN GRAPHS

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ABSTRACT

In this paper, we introduce the edge-to-vertex $e - u$ path, the edge-to-vertex detour distance $D(e, v)$, the edge-to-vertex $e - v$ detour, the edge-to-vertex detour eccentricity $e_{D2}(v)$, the edge-to-vertex detour radius R_2 , and the edge-to-vertex detour diameter D_2 of a connected graph G, where v is a vertex and e an edge in G. We determine these parameters for some standard graphs. It is shown that $R_2 \leq D_2 \leq 2R_2+1$ for every connected graph G and that every two positive integers a and b with $a \leq b \leq 2a+1$ are realizable as the edge-to-vertex detour radius and the edge-to-vertex detour diameter, respectively, of some connected graph. Also it is shown that for any two positive integers a, b with $a \leq b$ are realizable as the edge-to-vertex radius and the edge-to-vertex detour radius, respectively, of some connected graph and also for any two positive integers a, b with $a \leq b$ are realizable as the edge-to-vertex diameter and the edge-to-vertex detour diameter, respectively, of some connected graph. Also we introduce the edge-to-vertex detour center $C_{D2}(G)$ and the edge-to-vertex detour periphery $P_{D2}(G)$. It is shown that the edge-to-vertex detour center of every connected graph does not lie in a single block.

Key words : distance, detour distance, edge-to-vertex detour distance.
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1 Introduction

By a graph G = (V, E) we mean a finite undirected connected simple graph. For basic graph theoretic terminologies, we refer to Chartrand and Zhang [4]. If X ⊆ V, then X is the subgraph induced by X. For example if one is locating an emergency facility like police station, fire station, hospital, school, college, library, ambulance depot, emergency care center, etc., then the primary aim is to minimize the distance between the facility and the location of a possible emergency.

In 1964, Hakimi [6] considered the facility location problems as vertex-to-vertex distance in graphs. For any two vertices u and v in a connected graph G, the distance d(u, v) is the length of a shortest u − v path in G. For a vertex v in G, the eccentricity e(v) of v is the distance between v and a vertex farthest from v in G. The minimum eccentricity among the vertices of G is its radius and the maximum eccentricity is its diameter, denoted by rad(G) and diam(G) respectively. A vertex v in G is a central vertex if e(v) = rad(G) and the subgraph induced by the central vertices of G is the center Cen(G) of G. A vertex v in G is a peripheral vertex if e(v) = diam(G) and the subgraph induced by the peripheral vertices of G is the periphery Per(G) of G. If every vertex of G is a central vertex then G is called self-centered graph.

For example if one is making an election canvass or circular bus service the distance from the location is to be maximized. In 2005, Chartrand et. al. [3] introduced and studied the concepts of detour distance in graphs. For any two vertices u and v in a connected graph G, the detour distance D(u, v) is the length of a longest u − v path in G. For a vertex v in G, the detour eccentricity $e_D(v)$ of v is the detour distance between v and a vertex farthest from v in G. The minimum detour eccentricity among the vertices of G is its detour radius and the maximum detour eccentricity is its detour diameter, denoted by $rad_D(G)$ and $diam_D(G)$ respectively. The detour center, the detour self-centered and the detour periphery of a graph are defined similar to the center, the self-centered and the periphery of a graph, respectively.

For example when a railway line, pipe line or highway is constructed, the distance between the respective structure and each of the communities to be served is to be minimized. In a social network an edge represents two individuals having a common interest. Thus the centrality with respect to edges have interesting applications in social networks. In 2010, Santhakumaran [9] introduced the facility locational problem as edge-to-vertex distance in graphs as follows: For an edge e and a vertex v in a connected graph G, the edge-to-vertex distance is defined by $d(e, v) = \min\{d(u, v) : u \in e\}$. The edge-to-vertex eccentricity of e is defined by $e_2(e) = \max\{d(e, v) : v \in V\}$. A vertex v of G such that $e_2(e) = d(e, v)$ is called an edge-to-vertex eccentric vertex of v. The edge-to-vertex radius r_2 of G is defined by $r_2 = \min\{e_2(e) : e \in E\}$ and the edge-to-vertex diameter d_2 of G is defined by $d_2 = \max\{e_2(e) : e \in E\}$. An edge e for which $e_2(e)$ is minimum is called an edge-to-vertex central edge of G and the set of all edge-to-vertex central edges of G is the edge-to-vertex center $C_2(G)$ of G. An edge e for which $e_2(e)$ is maximum is called an edge-to-vertex peripheral edge of G and the set of all edge-to-vertex peripheral edges of G is the edge-to-vertex periphery P_2 (G) of G. If every edge of G is an edge-to-vertex central edge then G is called the edge-to-vertex self-centered graph.

These motivated us to introduce a distance called the edge-to-vertex deotur distance in graphs and investigate certain results related to edge-to-vertex detour distance and other distances in graphs. These ideas have interesting applications in channel assignment problem in radio technologies. Also there are useful applications of these concepts to security based communication network design. Throughout this paper, G denotes a connected graph with at least two vertices.

2 Edge-To-Vertex Detour Distance

Definition 2.1. Let e be an edge and v a vertex in a connected graph G. An edge-to-vertex e − v path P is a u − v path, where u is a vertex in e such that P contains no vertices of e other than u. The edge-to-vertex detour distance D(e, v) is the length of a longest e − v path in G. An e − v path of length D(e, v) is called an edge-to-vertex e − v detour or simply e − v detour. For our convenience an e − v path of length d(e, v) is called an edge-to-vertex e − v geodesic or simply e − v geodesic.

Example 2.2. Consider the graph G given in Fig 2.1. For the vertex v and the edge e = {u, w} in G, the paths P_1 : w, v; P_2 : u, z, r, v; P_3 : u, t, s, x, z, r, v and P_4 : u, t, s, x, y, z, r, v are e − v paths, while the paths Q_1 : u, w, v and Q_2 : w, u, z, r, v are not e− v paths. Now the edge-to-vertex distance d(e, v) = 1 and the edge-to-vertex detour distance D(e, v) = 7. Also P_1 is an e − v geodesic and P_4 is an e − v detour. Note that the e − u and e − w paths are trivial.

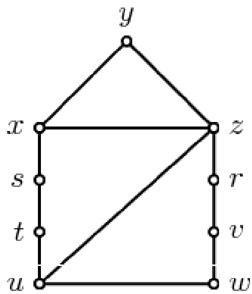

Fig 2.1: G

Since the length of an e − v path between an edge e and a vertex v in a graph G of order n is atmost n − 2, we have the following theorem.
Theorem 2.3. For any edge e and a vertex v in a non-trivial connected graph G of order n, 0 ≤ d(e, v) ≤ D(e, v) ≤ n − 2.

Remark 2.4. The bounds in the Theorem 2.3 are sharp. For any edge e and a vertex v in G, d(e, v) = D(e, v) = 0 if and only if v ∈ e and if G is a path P : $u_1, u_2, \dots, u_{n-1}$, un of order n, then d(e, v) = D(e, v) = n−2, where e = $\{u_1, u_2\}$and v = u_n. Also we note that if G is a tree, then d(e, v) = D(e, v) and if e is an edge and v ∈ e is a vertex in an even cycle, then d(e, v) < D(e, v).

Theorem 2.5. Let $K_{n,m}$ (n < m) be a complete bipartite graph with the partition V_1, V_2 of V ($K_{n,m}$) such that $|V_1|$ = n and $|V_2|$ = m. Let e be an edge and v a vertex such that v ∈ e in $K_{n,m}$, then

$$D(e,v) = \begin{cases} 2n-2, & if\ \ v \in V_1 \\ 2n-1 & if\ \ v \in V_2 \end{cases}$$

Proof. For an edge e and a vertex v ∈ e, the length of a longest e − v path is 2n− 2 if v ∈ V_1 and that of the e − v path is 2n − 1 if v ∈ V_2.

Corollary 2.6. Let v be a vertex and e an edge in a complete bipartite graph $K_{n,n}$ such that v ∈ e, then D(e, v) = 2n − 2.

Since every tree has unique e − v path between an edge e and a vertex v, we have the following theorem.

Theorem 2.7. If G is a tree, then d(e, v) = D(e, v) for every edge e and a vertex v in G.

The converse of the Theorem 2.7 is not true. For any edge e and a vertex v in K_3, d(e, v) = D(e, v) = 1 if v ∈ e and d(e, v) = D(e, v) = 0 if v ∈ e.

3 Edge-to-Vertex Detour Center

Definition 3.1. The edge-to-vertex detour eccentricity $e_{D2}(e)$ of an edge e in a connected graph G is defined as $e_{D2}(e)$ = max {D(e, v) : v ∈ V}. A vertex v for which $e_{D2}(e)$ = D(e, v) is called an edge-to-vertex detour eccentric vertex of e. The edge-to-vertex detour radius of G is defined as, R_2 = $rad_{D2}(G)$ = min {$e_{D2}(e)$: e ∈ E} and the edge-to-vertex detour diameter of G is defined as, D_2 = $diam_{D2}(G)$ = max {$e_{D2}(e)$: e ∈ E}. An edge e in G is called an edge-to-vertex detour central edge if $e_{D2}(e)$ =

R_2 and the edge-to-vertex detour center of G is defined as, C_{D2} (G) = Cen_{D2}(G) = {e ∈ E : e_{D2}(e) = R_2} . An edge e in G is called an edge-to-vertex detour peripheral edge if e_{D2}(e) = D_2 and the edge-to-vertex detour periphery of G is defined as, P_{D2}(G) = Per_{D2}(G) = {e ∈E : e_{D2} (e) = D_2} . If every edge of G is an edge-to-vertex detour central edge, then G is called an edge-to-vertex detour self centered graph. If G is the edge-to-vertex detour self-centered graph then G is called the edge-to-vertex detour periphery.

Example 3.2. For the connected graph G given in Fig. 3.1, the set of all edges in G are given by, E = {e_1 = {v_1 , v_2}, e_2 = {v_1 , v_3}, e_3 = {v_2 , v_3}, e_4 = {v_3 , v_4}, e_5 = {v_2 , v_4}, e_6 = {v_4 , v_5}}.

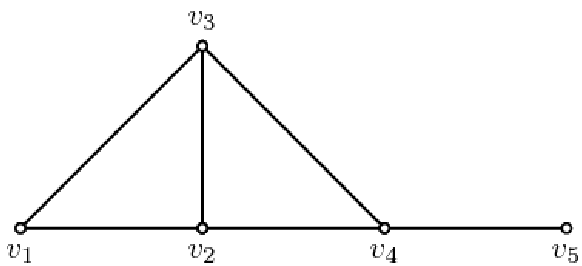

Fig. 3.1: G

The edge-to-vertex eccentricity e_2(e), the edge-to-vertex detour eccentricity e_{D2}(e) of all the edges of G are given in Table 1.

e	e_1	e_2	e_3	e_4	e_5	e_6
$e_2(e)$	2	2	2	1	1	2
$e_{D_2}(e)$	3	3	2	2	2	3

Table 1

The edge-to-vertex detour eccentric vertex of all the edges of G are given in Table 2.

Edge e	*Edge-to-Vertex Detour Eccentric vertex v*
e_4, e_5, e_6	v_1
e_4, e_6	v_2
e_5, e_6	v_3
e_1, e_2, e_3	v_5

Table 2

The edge-to-vertex radius $r_2 = 1$, the edge-to-vertex diameter $d_2 = 2$, the edge-to-vertex detour radius $R_2 = 2$ and the edge-to-vertex detour diameter $D_2 = 3$. Also the edge-to-vertex center C_2 (G) = $\{e_4 , e_5\}$, the edge-to-vertex periphery P_2 (G) = $\{e_1, e_2, e_3, e_6\}$, the edge-to-vertex detour center $C_{D2}(G) = \{e_3, e_4, e_5\}$ and the edge-to-vertex detour periphery $P_{D2}(G) = \{e_1, e_2, e_6\}$.

Example 3.3. The complete graph K_n, the cycle C_n, the wheel W_n and the complete bipartite graph $K_{n,n}$ are the edge-to-vertex detour self centered graphs.

Remark 3.4. An edge-to-vertex self-centered (periphery) graph need not be an edge-to-vertex detour self-centered (periphery) graph. For the graph G given in Fig 3.2, C_2 (G) = E(G), C_{D2} (G) = $\{f_3\}$, P_2 (G) = E(G) and P_{D2} (G) = $\{f_1, f_2, f_4, f_5\}$.

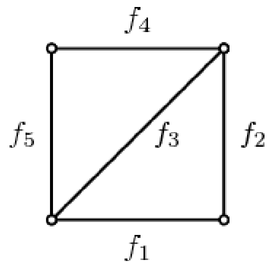

Fig. 3.2: G

The edge-to-vertex detour radius R_2 and the edge-to-vertex detour diameter D_2 of some standard graphs are given in Table 3.

G	K_n	P_n	$C_n (n \geq 4)$	$W_n (n \geq 4)$	$K_{n,m} (m \geq n)$
R_2	$n-2$	$\lfloor \frac{n-2}{2} \rfloor$	$n-2$	$n-2$	$\begin{cases} 2(n-1), & \text{if } n = m \\ 2n-1 & \text{if } n > m \end{cases}$
D_2	$n-2$	$n-2$	$n-2$	$n-2$	$\begin{cases} 2(n-1), & \text{if } n = m \\ 2n-1 & \text{if } n > m \end{cases}$

The following theorem is a consequence of Theorem 2.3.

Theorem 3.5. Let G be a connected graph. Then

(i) $0 \leq e_2 (e) \leq e_{D2} (e) \leq n - 2$ for every edge e in G.

(i) $0 \leq r_2 \leq R_2 \leq n - 2$.

(ii) $0 \leq d_2 \leq D_2 \leq n - 2$.

Remark 3.6. The bounds in the Theorem 3.5 (i) are sharp. If $G = K_2$, then $e_2(e) = e_{D2}(e) = 0$ for every edge e in G and if G is a path $P : u_1, u_2, \ldots, u_{n-1}, u_n$ of order n, then $e_2(e) = e_{D2}(e) = n - 2$, where $e = \{u_1 , u_2\}$ or $e = \{u_n , u_{n-1}\}$. Also we note that if G is a tree, then $e_2(e) = e_{D2}(e)$ for every edge e in G and for the graph G given in Fig. 2.1, $0 < e_2(e) < e_{D2}(e) < n - 2$, where e = {u, z}.

Theorem 3.7. For every connected graph G, $R_2 \leq D_2 \leq 2R_2+1$.

Proof. By definition $R_2 \leq D_2$. Now let $P : u_1, u_2, \ldots, u_{n-1}, u_n = v$ be an edge-to-vertex diametral path of length D_2 connecting an edge e and a vertex v, where $e = \{u_1, u_2\}$, so that $D_2 = D(e, v) = D(u_2, v)$ and let f be a edge of G such that $e_{D2}(f) = R_2 = D(y, u_n) = D(x, u_1)$, where f = {x, y}. It follows that $D_2 = D(e, v) \leq D(e, x)+D(x, y)+D(y, u_n) \leq R_2+1+R_2 \leq 2R_2 + 1$.

Remark 3.8. The bounds in the Theorem 3.7 are sharp. For the graph G given in Fig 3.3, it is easy to verify that $R_2 = 1$ and $D_2 = 3$.

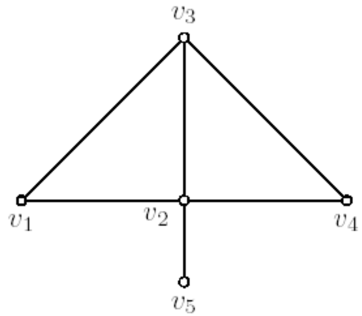

Fig. 3.3: G

Ostrand [8] showed that every two positive integers a and b with $a \leq b \leq 2a$ are realizable as the radius and diameter respectively of some connected graph and Chartrand et. al. [3] showed that every two positive integers a and b with $a \leq b \leq 2a$ are realizable as the detour radius and detour diameter respectively of some connected graph. Now we have a realization theorem for the edge-to-vertex detour radius and the edge-to-vertex detour diameter of some connected graph.

Theorem 3.9. For each pair a, b of positive integers with $a \leq b \leq 2a+1$, there exists a connected graph G with $R_2 = a$ and $D_2 = b$.

Proof. Case 1. a = b. Let $G = C_{a+2} : u_1, u_2, \ldots, u_{a+2}, u_1$ be a cycle of order a + 2. Then $e_{D2}(u_i u_{i+1}) = a$ for $1 \leq i \leq a + 1$. Thus $R_2 = a$ and $D_2 = b$ as a = b.

Case 2. $b \leq 2a$. Let $C_{a+2} : u_1, u_2, \ldots, u_{a+2}, u_1$ be a cycle of order a + 2 and $P_{b-a+1} : v_1, v_2, \ldots, v_{b-a+1}$ be a path of order b − a + 1. We construct the graph G of order b + 2 by identifying the vertex u_1 of C_{a+2} and v_1 of P_{b-a+1} as shown in Fig. 3.4. It is easy to verify that $e_{D2}(u_1 u_2) = e_{D2}(u_1 u_{a+2}) = a$. Also $e_{D2}(u_i u_{i+1}) = b-i+2$ for $2 \leq i \leq \lceil (a+2)/2 \rceil$ and $e_{D2}(u_i u_{i+1}) = b-a+i-1$ for $\lceil (a+2)/2 \rceil < i \leq a+1$. Also $e_{D2}(v_i v_{i+1}) = a+i$ for $1 \leq i \leq b - a$. In particular, $e_{D2}(u_2 u_3) = e_{D2}(u_{a+1} u_{a+2}) = e_{D2}(v_{b-a} v_{b-a+1}) = b$. It is easy to verify that there is no edge e in G with $e_{D2}(e) < a$ and there is no edge e′ in G with $e_{D2}(e') > b$. Thus $R_2 = a$ and $D_2 = b$ as $a < b \leq 2a$.

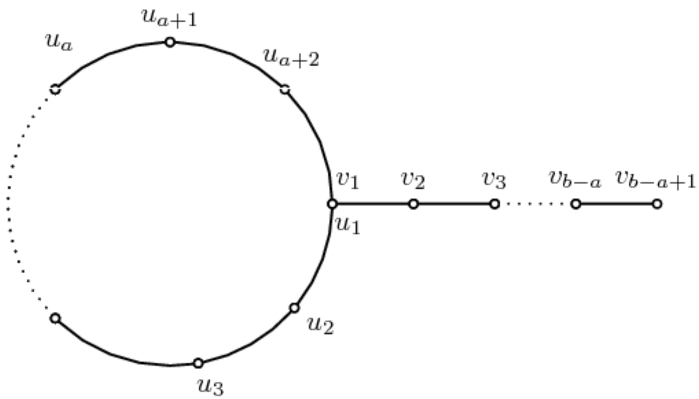

Fig. 3.4: G

Case 3. $b = 2a + 1$. Construct the graph G as shown in Fig 3.5, it is easy to verify that $e_{D2}(xv_1) = a$ and $e_{D2}(v_{b-a-1}v_{b-a}) = b$. Also there is no edge e in G with $e_{D2}(e) < a$ and there is no edge e′ in G with $e_{D2}(e') > b$. Thus $R_2 = a$ and $D_2 = b$ as $b = 2a+1$.

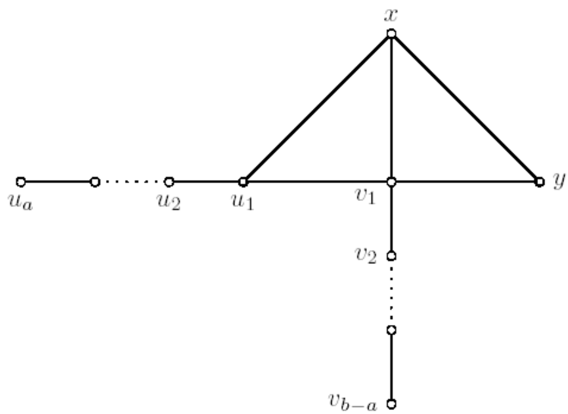

Fig. 3.5: G

Chartrand et. al. [3] showed that every pair a, b of positive integers with $a \leq b$ is realizable as the radius and the detour radius of some connected graph. Now we have a realization theorem for the edge-to-vertex radius and the edge-to-vertex detour radius of some connected graph.

Theorem 3.10. For each pair a, b of positive integers with $a \leq b$, there exists a connected graph G such that $r_2 = a$ and $R_2 = b$.

Proof. Case 1. $a = b$. Let $P_1 : u_1, u_2, \ldots, u_a, u_{a+1}$ and $P_2 : v_1, v_2, \ldots, v_a, v_{a+1}$ be two paths of order $a + 1$. We construct the graph G of order 2a

+ 2 by joining u_1 in P_1 and v_1 in P_2 by an edge. Then $e_2(u_1 v_1) = e_{D2}(u_1 v_1) = a$ and $e_2(u_i u_{i+1}) = e_2(v_i v_{i+1}) = a + i$ for $1 \leq i \leq a$. It is easy to verify that there is no edge e in G with $e_2(e) = e_{D2}(e) < a$. Thus $r_2 = a$ and $R_2 = b$ as $a = b$.

Case 2. $a < b$. We have the following two subcases:

Subcase 1 of Case 2. $a = 1$. Any complete graph of order K_{b+2} is the desired graph.

Subcase 2 of Case 2. $a \geq 2$. Let $P_1 : u_1, u_2, \ldots, u_a, u_{a+1}$ and $Q_1 : v_1, v_2, \ldots, v_a, v_{a+1}$ be two paths of order $a + 1$. Let $P_2 : w_1, w_2, \ldots, w_{b-a+2}$ and $Q_2 : z_1, z_2, \ldots, z_{b-a+2}$ be two paths of order $b - a + 2$. We construct the graph G of order $2b + 2$ as follows: (i) identify the vertices u_1 in P_1 with w_1 in P_2 and also identify the vertices v_1 in Q_1 with z_1 in Q_2 (ii) identify the vertices u_3 in P_1 with w_{b-a+2} in P2 and also identify the vertices z_{b-a+2} in Q_2 with v_3 in Q_1 (iii) join each vertex w_i $(2 \leq i \leq b - a + 1)$ in P_2 with u_2 in P_1 and join each vertex z_i $(2 \leq i \leq b - a + 1)$ in Q_2 with v_2 in Q_1 (iv) join u_1 in P_1 with v_1 in Q_1 . The resulting graph G is shown in Fig. 3.6.

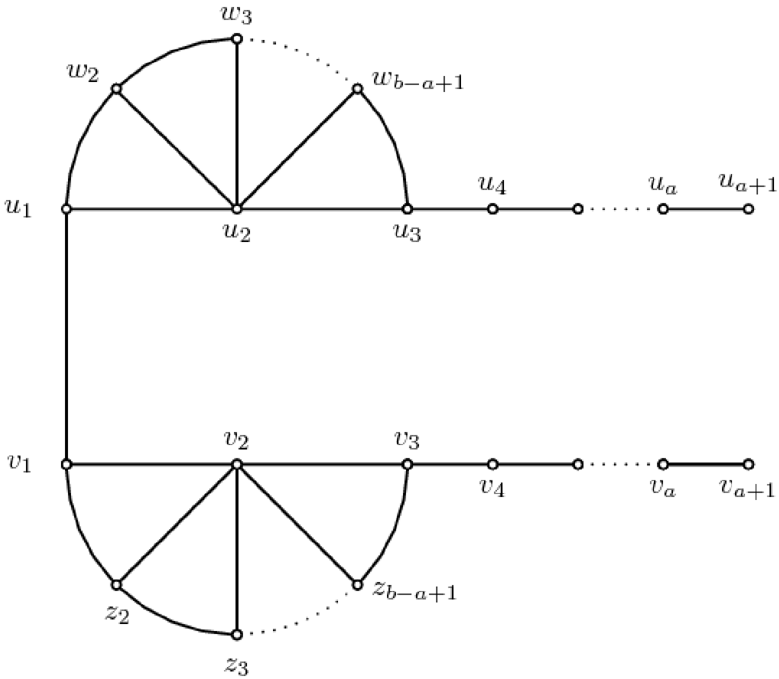

Fig 3.6: G

It is easy to verify that

$e_2(u_1v_1) = a$

$e_2(u_iu_{i+1}) = a + i \text{ if } 1 \leq i \leq a$

$e_2(v_iv_{i+1}) = a + i \text{ if } 1 \leq i \leq a$

$$e_2(w_iw_{i+1}) = \begin{cases} a+1, & \text{if } i = 1 \\ a+2, & \text{if } i = 2 \\ a+3, & \text{if } 3 \leq i \leq b-a+1 \end{cases}$$

$$e_2(z_iz_{i+1}) = \begin{cases} a+1, & \text{if } i = 1 \\ a+2, & \text{if } i = 2 \\ a+3, & \text{if } 3 \leq i \leq b-a+1 \end{cases}$$

$e_2(u_2w_i) = a + 2 \text{ if } 1 \leq i \leq b-a+1$

$e_2(v_2z_i) = a + 2$ if $1 \leq i \leq b - a + 1$
$e_{D_2}(u_1v_1) = b$

$$e_{D_2}(u_iu_{i+1}) = \begin{cases} b+1, & \text{if } i = 1 \\ 2b - a + i, & \text{if } 2 \leq i \leq a \end{cases}$$

$$e_{D_2}(v_iv_{i+1}) = \begin{cases} b+1, & \text{if } i = 1 \\ 2b - a + i, & \text{if } 2 \leq i \leq a \end{cases}$$

$$e_{D_2}(w_iw_{i+1}) = \begin{cases} b+1, & \text{if } i = 1 \\ 2b - a + 2, & \text{if } 2 \leq i \leq b - a + 1 \end{cases}$$

$$e_{D_2}(z_iz_{i+1}) = \begin{cases} b+1, & \text{if } i = 1 \\ 2b - a + 2, & \text{if } 2 \leq i \leq b - a + 1 \end{cases}$$

$e_{D_2}(u_2w_i) = b + i$ if $1 \leq i \leq b - a + 1$
$e_{D_2}(v_2z_i) = b + i$ if $1 \leq i \leq b - a + 1$

It is easy to verify that there is no edge e in G with $e_2(e) < a$ and $e_{D2}(e) < b$. Thus $r_2 = a$ and $R_2 = b$ as $a < b$.

Chartrand et. al. [3] showed that every pair a, b of positive integers with a ≤ b is realizable as the diameter and the detour diameter of some connected graph. Now we have a realization theorem for the edge-to-vertex diameter and the edge-to-vertex detour diameter of some connected graph.

Theorem 3.11. For any two positive integers a, b with a ≤ b, there exists a connected graph G such that $d_2 = a$ and $D_2 = b$.

Proof. Case 1. a = b. Let $P_{a+2} : u_1, u_2, \ldots, u_a, u_{a+1}, u_{a+2}$ be a path of order a + 2. Then $e_2(u_i u_{i+1}) = e_{D2}(u_i u_{i+1}) = a - i + 1$ for $1 \leq i \leq \lceil (a+1)/2 \rceil$ and $e_2(u_i u_{i+1}) = e_{D2}(u_i u_{i+1}) = i - 1$ for $\lceil (a+1)/2 \rceil < i \leq a +$

1. In particular $e_2(u_1u_2) = e_{D2}(u_1u_2) = e_2(u_{a+1}u_{a+2}) = e_{D2}(u_{a+1}u_{a+2}) =$ a. It is easy to verify that there is no edge e in G with $e_2(e) = e_{D2}(e) >$ a. Thus $d_2 = a$ and $D_2 = b$ as $a = b$.

Case 2. $a < b$. We have the following two subcases:

Subcase 1 of Case 2. $a = 1$. Any complete graph of order K_{b+2} is the desired graph.

Subcase 2 of Case 2. $a = 2$. Let G be the graph obtained by joining any one vertex of the complete graph K_b of order b with any vertex of a path $P_3 : x_1, x_2, x_3$ of order 3. It is easy to verify that $e_2(x_2x_3) = a$ and $e_{D2}(x_2x_3) = b$. Also there is no edge e in G with $e_2(e) > a$ and $e_{D2}(e) > b$.

Subcase 3 of Case 2. $a \geq 3$. Let $P_1 : u_1, u_2, \ldots, u_a, u_{a+1}$ be a path of order $a + 1$. Let $P_2 : w_1, w_2, \ldots, w_{b-a+2}$ be a path of order $b - a + 2$. Let $P_3 : x_1, x_2$ be a path of order 2. We construct the graph G of order $b + 2$ as follows: (i) identify the vertices u_1 in P_1, w_1 in P_2 with x_1 in P_3 and identify the vertices u_3 in P_1 with w_{b-a+2} in P_2 (ii) join each vertex w_i $(2 \leq i \leq b - a + 1)$ in P_2 with u_2 in P_1. The resulting graph G is shown in Fig. 3.7.

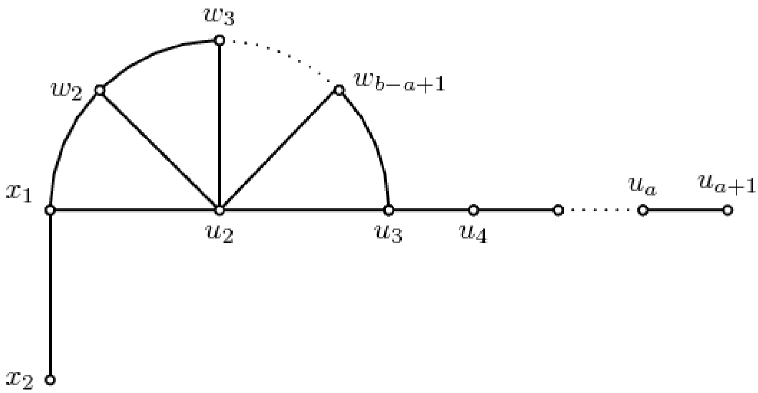

Fig 3.7: G

It is easy to verify that

$$e_{D_2}(w_iw_{i+1}) = \begin{cases} b-1, & \text{if } 1 \le i \le b-a \\ b-a+2, & \text{if } i = b-a+1 \text{ for } b-a+2 \ge a-2 \\ a-2, & \text{if } i = b-a+1 \text{ for } b-a+2 \le a-2 \end{cases}$$

$$e_2(x_1x_2) = a$$

$$e_{D_2}(x_1x_2) = b$$

$$e_2(u_iu_{i+1}) = \begin{cases} a-i, & \text{if } 1 \le i \le \left\lfloor \frac{a}{2} \right\rfloor \\ i, & \text{if } \left\lfloor \frac{a}{2} \right\rfloor < i \le a \end{cases}$$

$$e_{D_2}(u_iu_{i+1}) = \begin{cases} b-1, & \text{if } i = 1 \\ b-a+i, & \text{if } 2 \le i \le a \text{ for } b-a+i \ge a-i \\ a-i, & \text{if } 2 \le i \le a \text{ for } b-a+i \le a-i \end{cases}$$

$$e_2(u_2w_i) = a-1 \text{ if } 2 \le i \le b-a+1$$

$$e_{D_2}(u_2w_i) = \begin{cases} b-i, & \text{if } 1 \le i \le b-a+1 \text{ for } b-i \ge i \\ i, & \text{if } 1 \le i \le b-a+1 \text{ for } b-i \le i \end{cases}$$

$$e_2(w_iw_{i+1}) = \begin{cases} a, & \text{if } 1 \le i \le b-a-1 \\ a-1, & \text{if } i = b-a \text{ for } a-1 \ge a \\ a, & \text{if } i = b-a \text{ for } a-1 \le a \\ a-2, & \text{if } i = b-a+1 \text{ for } a-2 \ge a \\ a, & \text{if } i = b-a+1 \text{ for } a-2 \le a \end{cases}$$

$$e_{D_2}(w_iw_{i+1}) = \begin{cases} b-1, & \text{if } 1 \le i \le b-a \\ b-a+2, & \text{if } i = b-a+1 \text{ for } b-a+2 \ge a-2 \\ a-2, & \text{if } i = b-a+1 \text{ for } b-a+2 \le a-2 \end{cases}$$

It is easy to verify that there is no edge e in G with $e_2(e) > a$ and $e_{D2}(e) > b$. Thus $d_2 = a$ and $D_2 = b$ as $a < b$.

Problem 3.12. Characterize the graphs such that $C_{D2}(G) = C_2(G)$

Problem 3.13. Characterize the graphs such that $P_{D2}(G) = P_2(G)$

Problem 3.14. Characterize the graphs such that $C_{D2}(G) = P_{D2}(G)$

Problem 3.14. Is every graph an edge-to-vertex detour center of some graph?

Remark 3.15. The edge-to-vertex detour center of every connected graph does not lie in a single block of G. For the Path P_{2n+1} of order 2n + 1, the edge-to-vertex detour center is always P_3, which does not lie in a single block.

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